

Complete, fold and seal
(see note 2 below)

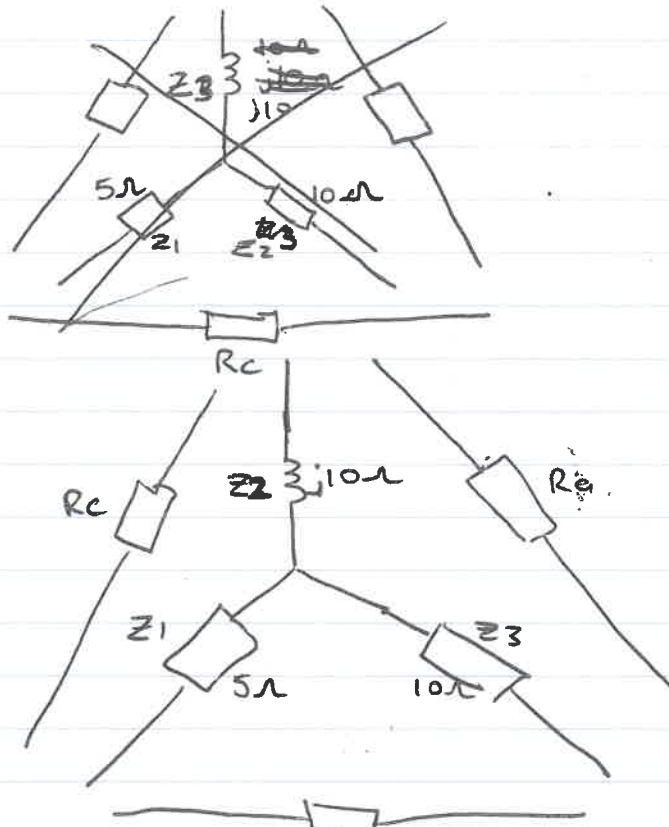
FORENAME(S): Jack

12 Pages

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
7. If more books than one are used, fasten them together using the tags available from the invigilator(s).
8. Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.

[illegible]

a)



$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{Z_1} \quad \checkmark$$

$$R_a = \frac{(5 \times 10) + (10 \times j10) + (j10 \times 5)}{5} = \frac{50 + j100 + j50}{5} \quad \checkmark$$

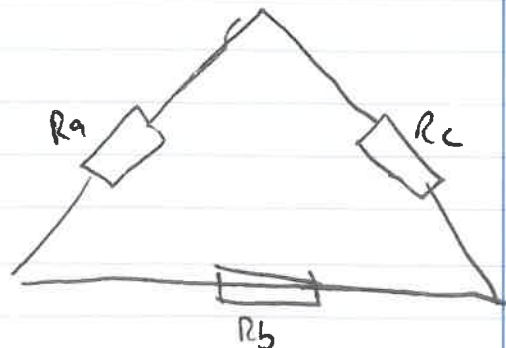
$$R_a = \frac{50 + j150}{5} = 10 + j30 \, \Omega \quad \checkmark$$

$$R_b = \frac{R_T}{Z_2} = \frac{50 + j150}{j10} \cdot \frac{-j10}{-j10} = \frac{-j500 + 1500}{100} \quad \checkmark$$

$$R_b = 15 - j5 \, \Omega \quad \checkmark$$

$$R_c = \frac{R_T}{Z_3} = \frac{50 + j150}{10} \quad \checkmark$$

$$R_c = 5 + j15 \, \Omega \quad \checkmark$$



QL on S11

b) $V^{th} = \frac{3+j4}{5+3+j4} \times 10 \angle 0^\circ V$ ✓

$$= \frac{3+j4}{8+j4} \times 10$$

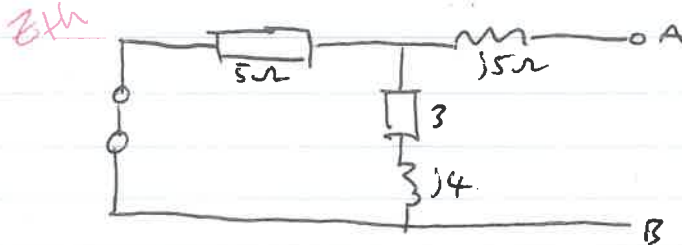
$$= \frac{3+j4}{8+j4} \frac{8-j4}{8-j4} = \frac{24-j12+j32+16}{64+16}$$

$$= \frac{40+j20}{80} \times 10$$

$$= (0.5 + j0.25) \times 10 \checkmark$$

$$V^{th} = 5 + j2.5 V \checkmark$$

R^{th} → Disconnect the ~~load~~ voltage source.



$R^{th} = \frac{5 \times (3+j4)}{5+3+j4} = \frac{15+j20}{8+j4} + (j5)$

$$= j5 + \frac{15+j20}{8+j4} \frac{8-j4}{8-j4}$$

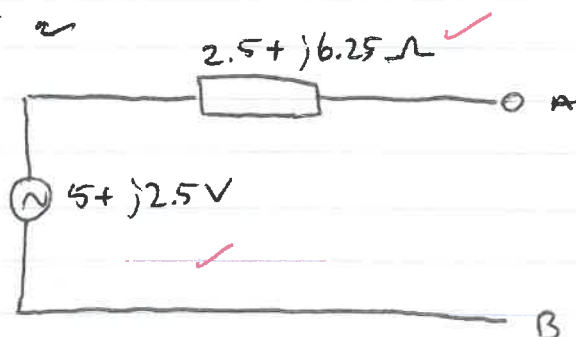
$$= \frac{120 - j60 + j160 + 80}{64 + 16} = \frac{200 + j100}{80} + j5$$

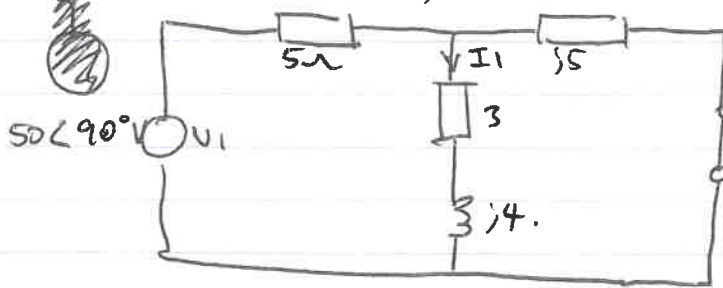
Z_{th}

$R^{th} = 2.5 + j1.25 + j5 \Omega$

$$= 2.5 + j6.25 \Omega \checkmark$$

Thevenin Equivalent ⇒



c) Disconnect V_2 ;

$$Z_T = \frac{j5 \times (3 + j4)}{j5 + 3 + j4} (+5) \checkmark$$

$$= \frac{-20 + j15}{3 + j9} \cdot \frac{3 - j9}{3 - j9} = \frac{-60 + j180 + j45 + 135}{9 + 81}$$

$$= \frac{75 + j225}{90} (+5) \checkmark$$

$$= 0.83 + j2.5 \checkmark$$

$$Z_T = 5.83 + j2.5 \Omega \checkmark$$

$$I_T = \frac{V_1}{Z_T} = \frac{50 \angle 90^\circ}{5.83 + j2.5}$$

$$\text{Re}(50, 90) = j50$$

$$= \frac{j50}{5.83 + j2.5} = \frac{5.83 - j2.5}{5.83 - j2.5} \cdot \frac{j291.5 + 125}{34 + 6.25}$$

$$I_T = \frac{125 + j291.5}{40.25} \checkmark$$

$$I_T = 3.1 + j7.2 \text{ A} \checkmark$$

$$I_1 = \frac{j5(3.1 + j7.2)}{3 + j9} = \frac{-36 + j15.5}{3 + j9} \cdot \frac{3 - j9}{3 - j9}$$

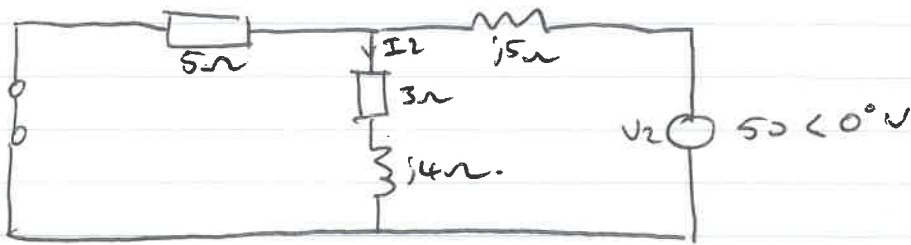
$$= \frac{-108 + j324 + j46.5 + 139.5}{90}$$

$$I_1 = \frac{31.5 + j370.5}{90} = 0.35 + j4.1 \text{ A} \checkmark$$

Q1
C

14

Disconnect V_1 .



$$Z_T = \frac{5(3+j4)}{5+3+j4} = \frac{15+j20}{8+j4} \cdot \frac{8-j4}{8-j4}$$

$$= \frac{120 - j60 + j160 + 80}{64 + 16}$$

$$= \frac{200 + j100}{80} + j5 = 2.5 + j1.25 + j5$$

$$Z_T = 2.5 + j6.25 \Omega$$

$$I_T = \frac{V_2}{Z_T}$$

$$= \frac{50}{2.5 + j6.25} \cdot \frac{2.5 - j6.25}{2.5 - j6.25} = \frac{125 - j312.5}{6.25 + 39}$$

$$I_T = \frac{125 - j312.5}{45.25}$$

$$= 2.76 - j6.9 \text{ A}$$

$$I_2 = \frac{5(2.76 - j6.9)}{8+j4} = \frac{13.8 - j34.5}{8+j4}$$

$$= \frac{13.8 - j34.5}{8+j4} \cdot \frac{8-j4}{8-j4} = 110.4$$

$$= \frac{110.4 - j55.2 - j276 - 138}{80}$$

$$\frac{-27.6 - j331}{80}$$

$$I_2 = -0.345 - j4.17 \text{ A}$$

$$I = -0.34 - j4.14 + 0.35 + j4.1 \quad \times$$

$$I = 0.01 - j0.04 \quad \times$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx] \quad \checkmark$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx \quad \checkmark$$

$$= \frac{1}{2\pi} \int_{-\pi}^0 f(x) dx + \frac{1}{2\pi} \int_0^{\pi} f(x) dx \quad \checkmark$$

$$= \frac{1}{2\pi} \int_{-\pi}^0 -3 dx + \frac{1}{2\pi} \int_0^{\pi} 3 dx \quad \checkmark$$

$$= \frac{-3}{2\pi} \left[x \right]_{-\pi}^0 + \frac{3}{2\pi} \left[x \right]_0^{\pi} \quad \checkmark$$

$$= \frac{-3}{2\pi} [0 - (-\pi)] + \frac{3}{2\pi} [\pi - 0] \quad \checkmark$$

$$= \frac{-3}{2\pi} [\pi] + \frac{3}{2\pi} [\pi] \quad \checkmark$$

$$a_0 = -1.5 + 1.5 \quad \checkmark$$

$$a_0 = 0 \quad \checkmark$$

8/11

Q225

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx \quad \checkmark$$

$$= \frac{1}{\pi} \int_{-\pi}^0 f(x) \cos nx \, dx + \frac{1}{\pi} \int_0^{\pi} f(x) \cos nx \, dx \quad \checkmark$$

$$= \frac{1}{\pi} \int_{-\pi}^0 -3 \cos nx \, dx + \frac{1}{\pi} \int_0^{\pi} 3 \cos nx \, dx \quad \checkmark$$

$$= \frac{-3}{\pi} \left[\frac{\sin nx}{n} \right]_{-\pi}^0 + \frac{3}{\pi} \left[\frac{\sin nx}{n} \right]_0^{\pi} \quad \checkmark$$

$$= \frac{-3}{n\pi} \left[\sin nx \right]_{-\pi}^0 + \frac{3}{n\pi} \left[\sin nx \right]_0^{\pi} \quad \checkmark$$

$$= \frac{-3}{n\pi} \left[\sin(0) - \sin(n\pi) \right] + \frac{3}{n\pi} \left[\sin(n\pi) - \sin(0) \right]$$

$$= \frac{-3}{n\pi} \left[0 - 0 \right] + \frac{3}{n\pi} \left[0 - 0 \right]$$

$$a_n = 0 \quad \checkmark \quad \text{forall}$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx \quad \checkmark$$

$$= \frac{1}{\pi} \int_{-\pi}^0 f(x) \sin nx \, dx + \frac{1}{\pi} \int_0^{\pi} f(x) \sin nx \, dx \quad \checkmark$$

$$= \frac{1}{\pi} \int_{-\pi}^0 -3 \sin nx \, dx + \frac{1}{\pi} \int_0^{\pi} 3 \sin nx \, dx \quad \checkmark$$

$$= \frac{1}{\pi} \left[-3 \frac{-\cos nx}{n} \right]_{-\pi}^0 + \frac{1}{\pi} \left[3 \frac{-\cos nx}{n} \right]_0^{\pi} \quad \checkmark$$

$$= \frac{3}{n\pi} \left[\cos nx \right]_{-\pi}^0 + \frac{-3}{n\pi} \left[\cos nx \right]_0^{\pi}$$

$$= \frac{3}{1\pi} [\cos(0) - \cos(-\pi)] + \frac{-3}{1\pi} [\cos(\pi) - \cos(0)]$$

$$= \frac{3}{1\pi} [1 - (-1)] + \frac{-3}{1\pi} [-1 - 1]$$

$$= \frac{3}{1\pi} [2] + \frac{-3}{1\pi} [-2]$$

$$= \frac{6}{1\pi} + \frac{6}{1\pi}$$

$$b_n \text{ when odd} = \frac{12}{1\pi}$$

b_n when even ;

$$= \frac{3}{1\pi} [\cos(0) - \cos(2\pi)] + \frac{-3}{1\pi} [\cos(2\pi) - \cos(0)]$$

$$= \frac{3}{1\pi} [1 - 1] + \frac{-3}{1\pi} [1 - 1]$$

$$= \frac{3}{1\pi} [0] + \frac{-3}{1\pi} [0]$$

$$b_n \text{ when even} = 0.$$

$$f(x) = \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx]$$

$$f(x) = \frac{12}{\pi} \sin x + \frac{12}{3\pi} \sin 3x + \frac{12}{5\pi} \sin 5x + \frac{12}{7\pi} \sin 7x$$

Q3 a) i) $1 + 2t - \frac{1}{3}t^4 = \frac{1}{s} + \frac{2}{s^2} - \frac{1}{3} \frac{4!}{s^5}$
 $= \frac{1}{s} + \frac{2}{s^2} - \frac{8}{s^5}$ ✓ ②

ii) $5e^{2t} - 3e^{-t} = \frac{5}{s-2} - \frac{3}{s+1}$ ✓
 $= \frac{5(s+1) - 3(s-2)}{(s-2)(s+1)}$ ✓
 $= \frac{5s+5-3s+6}{s^2+s-2s-2} = \frac{2s+11}{s^2-s-2}$ ✓ ②

iii) $6 \sin(3t) - 4 \cos(5t) = \frac{6 \times 3}{s^2+3^2} - \frac{4s}{s^2+5^2}$ ✓
 $= \frac{18}{s^2+9} - \frac{4s}{s^2+25}$ ✓ ②

iv) $e^{-2t} \sin(4t) = \frac{4}{(s+2)^2+16}$ ✓ ②

v) $\cos(2t) + 3e^{-4t} = \frac{s}{s^2+4} + \frac{3}{s+4}$ ✓ ②

b) $\frac{dy}{dt} + 2y = 12 \quad y=10, \quad t=0.$

$$L\left[\frac{df(t)}{dt}\right] = sF(s) - f(0).$$

$$sY(s) - y(0) + 2Y(s) = \frac{12}{s}$$
 ✓

$$sY(s) - 10 + 2Y(s) = \frac{12}{s}$$
 ✓

Q3
a)

10

Q3
b

10

$$s Y(s) + 2 Y(s) = \frac{12}{s} + 10 \quad \checkmark$$

$$Y(s) (s + 2) = \frac{12}{s} + 10 \quad \checkmark$$

$$Y(s) = \frac{12}{s(s+2)} + \frac{10}{s+2} \quad \checkmark$$

$$\frac{12}{s(s+2)} = \frac{A}{s} + \frac{B}{(s+2)} \quad \checkmark$$

$$\begin{aligned} 4 a) i) V_{RMS} &= \sqrt{V_0^2 + \sum_{n=1}^{\infty} \left(\frac{V_n^2}{2} \right)} \quad \checkmark \\ &= \sqrt{50^2 + \frac{20^2}{2} + \frac{10^2}{2}} \quad \checkmark \\ &= 33.9 \text{ V} \quad \checkmark \end{aligned}$$

$$\begin{aligned} ii) I_{RMS} &= \sqrt{I_0^2 + \sum_{n=1}^{\infty} \left(\frac{I_n^2}{2} \right)} \quad \checkmark \\ &= \sqrt{6^2 + \frac{4^2}{2} + \frac{2^2}{2}} \quad \checkmark \\ &= 6.48 \text{ A} \quad \checkmark \end{aligned}$$

$$\begin{aligned} iii) P_{av} &= \sum_{n=1}^{\infty} V_0 I_0 \cos(\phi - \theta) \quad \checkmark \\ &= V_0 I_0 \sum_{n=1}^{\infty} \frac{V_n I_n}{2} \cos(\phi - \theta) \quad \checkmark \end{aligned}$$

Difference between.

Q4

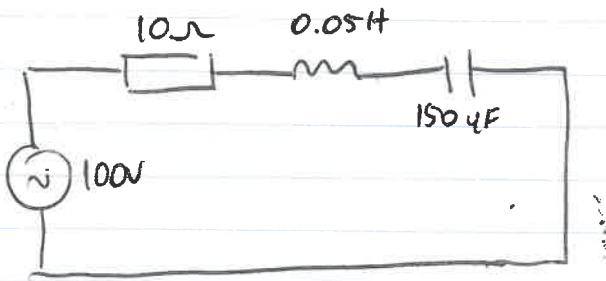
a

4

$$P_{av} = (30 \times 6) + 30 \times 2 \cos(50) \quad \times$$

$$= 218.57 \text{ W.} \quad \times$$

b).



50 Hz supply.

$$X_C = \frac{1}{j2\pi fC} \quad \checkmark$$

$$= \frac{1}{j2\pi \times 50 \times 150 \times 10^{-6}} \quad \checkmark$$

$$X_C = \cancel{+j21.2} - j21.2 \Omega \quad \checkmark \checkmark$$

$$X_L = j2\pi fL$$

$$= j2\pi \times 50 \times 0.05.$$

$$X_L = j15.7 \Omega \quad \checkmark \quad Bq$$

$$Z_T = R + X_L - X_C$$

$$= 10 + j15.7 - j21.2.$$

$$Z_T = 10 - j5.5 \Omega \quad \checkmark$$

$$I_T = \frac{V}{Z_T}$$

$$= \frac{100}{10 - j5.5} \cdot \frac{10 + j5.5}{10 + j5.5}.$$

$$= \frac{1000 + j550}{100 + 30.25} = \frac{1000 + j550}{130.25}.$$

$$I_T = 7.7 + j4.2 \text{ A} \quad \checkmark$$

$$\text{Magnitude} = \sqrt{7.7^2 + 4.2^2}$$

$$= 8.7 \text{ A} \quad \checkmark$$

$$\text{Phase angle} = \tan^{-1} \frac{4.2}{7.7}$$

$$\phi = 28.6^\circ \quad \checkmark$$

$$\text{Power } \cancel{P} = I^2 \times R.$$

$$= (7.7 + j4.2)^2 \times R.$$

~~7.7 + j4.2~~

$$(7.7 + j4.2)(7.7 + j4.2)$$

$$= 59.29 - 17.64$$

$$= 41.65 \quad \times$$

$$\text{Power} = 41.65 \times 10$$

$$= 416.5 \text{ W.}$$

$$\text{Power factor} = \cos(28.6)$$

$$\text{p.f.} = 0.88 \quad \checkmark \quad \textcircled{1}$$